Lesson 2A: Geometric Sequences, Explicit Formula

Classwork

**Example 1**

Consider the sequence 5, 8, 11, 14, 17, ….

* 1. Write the recursive formula and the explicit formula for the sequence.
	2. Explain how each part of the formula relates to the sequence.

**Example 2**

Now consider the sequence 5, 15, 45, 135,…

a. What are the next three terms in the sequence?

Consider the sequence that follows a “multiplication of 3” pattern: $4, 12, 36, 108, 324,$ ….

Consider the sequence that follows a “multiplication of$ \frac{1}{3}$ pattern: $33, 11, \frac{11}{3}, \frac{11}{9} , \frac{ 11}{27}, ….$

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| --- |
| An **geometric sequence** is a numerical pattern that increases or decreases by multiplying the previous term by a nonzero constant called the **common ratio.** |
| 1, -4, 16, -64, -256, …r = -4  | 266, 128, 64, 32, …$$r= \frac{1}{2}$$ |

Determine whether each sequence is a geometric sequence.

a. -4, -2, -1, - $\frac{ 1}{2}$, … b. 1, 4, 9, 25, ... c. $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, …$ d. -3, 15, -75, 375, …

Find the next three terms in the geometric sequence $ 4, 12, 36, 108, …$ .

Find the next four terms of the geometric sequence -4, 2, -1, $\frac{ 1}{2}$, … .

Each term in a geometric sequence can be expressed in terms of the **first term a1** and the **common ratio r**.

5, 15, 45, 135,…

|  |  |  |  |
| --- | --- | --- | --- |
| **Term** | **Symbol** | **In terms of a1 and d** | **Numbers** |
| first term | a1 | a1 | 5 |
| second term | a2 | a1r | 5(3) = |
| third term | a3 | a1r2 | 5(3)2 = |
| fourth term | a4 | a1r3 | 5(3)3 = |
| … | … | … | … |
| nth term | an | **a1r(n-1)** | 5r(n-1) |

Find the 15th term of the geometric sequence above.

Write an equation for the nth term of the geometric sequence $4, 12, 36, 108, …$

**Step 1: Find the common ratio.**

**Step 2: Write the equation.**

 **an = a1r(n-1)**

Find the 9th term:

Write an equation for the nth term of the geometric sequence $4, 12, 36, 108, …$

**Step 1: Find the common ratio.**

**Step 2: Write the equation.**

 **an = a1r(n-1)**

Find the 15th term:

Find the 25th term:

Write an equation for the nth term of the geometric sequence -4, 2, -1, $\frac{ 1}{2}$, … .

**Step 1: Find the common ratio.**

**Step 2: Write the equation.**

 **an = a1r(n-1)**

Find the 10th term:

Find the 20th term:

Lesson 2B: Geometric Sequences, Recursive Formula

Classwork

**Example 1**

What sequence does an+1 = an · 3 for $n\geq 1$ and a1 = 5 generate?

What sequence does an+1 = an ÷ 3 for $n\geq 1$ and a1 = 30 generate?

In a **geometric sequence** each term is found by multiplying a fixed number called the **common ratio(*r*)**, to the previous term. Example: 5, 15, 45, 135,…

The following equation is a recursive definition of a geometric sequence:

**an+1= an r**

|  |  |  |  |
| --- | --- | --- | --- |
| **Term** | **Symbol** | **In terms of a1 and d** | **Numbers** |
| first term | a1 | a1 | 5 |
| second term | a2 | a1 · r | 5 · 3 = |
| third term | a3 | a2 · r | 15 · 3 = |
| fourth term | a4 | a3 · r | 45 · 3 = |
| … | … | … | … |
| nth term | an | an-1 · r | an  |
| nth + 1 term | an+1 | an · r | an+1 |

**Example 1**

**For problems 1-3,the recursive formula is given list the first five terms of each sequence.**

1. $a\_{n+1}=a\_{n}· 6$, where $a\_{1}=11$ for $n\geq 1$

1. $a\_{n+1}=a\_{n}÷ $ 6, where $a\_{1}=36$ for $n\geq 1$

1. $a\_{n+1}=a\_{n}·4$, where $a\_{1}=3$ for $n\geq 1$

4. Write the recursive formula for the geometric sequence 64, 32, 16, 8, … , where a1 is 64. Find the next three terms.

5. Write the recursive formula for the geometric sequence -3, 15, -75, 275 … , where a1 is -3. Find the next four terms.

6. Determine the recursive formula for each of the following geometric sequences.

a. 24, 36, 54, 81, …. b. 9, 3, 1, $\frac{1}{3}$, ….

**Example 2**

1. Consider a sequence given by the formula $a\_{n+1}=a\_{n}∙-2$ where $a\_{1}=1$.

* 1. List the first five terms of the sequence.
	2. Write an explicit formula.
	3. Find $a\_{6}$ and $a\_{100}$ of the sequence.

2. Graph the first five terms of the geometric sequence 64, 32, 16, 8, … ,

|  |  |
| --- | --- |
| **an** | **an+1 = an ÷ 2** |
| a1 | 64 |
| a2 | 32 |
| a3 | 16 |
| a4 | 8 |
| a5 |  |



3. Graph the first five terms of the geometric sequence 3, 6, 12, 24,…

|  |  |
| --- | --- |
| **an** | **an+1 = an · 2** |
| a1 | 3 |
| a2 | 6 |
| a3 | 12 |
| a4 | 24 |
| a5 |  |



4. Find the 11th term if the sequence 3, -6, 12, -24, … .

Lesson Summary

RECURSIVE SEQUENCE (description). An example of a *recursive sequence* is a sequence that (1) is defined by specifying the values of one or more initial terms and (2) has the property that the remaining terms satisfy a recursive formula that describes the value of a term based upon an expression in numbers, previous terms, or the index of the term.

An explicit formula specifies the $n$th term of a sequence as an expression in $n$.

A recursive formula specifies the $n$th term of a sequence as an expression in the previous term (or previous couple of terms).

Problem Set 2B

For problems 1-7, list the first five terms of each sequence.

1. $a\_{n+1}=a\_{n}+6$, where $a\_{1}=11$ for $n\geq 1$
2. $a\_{n}=a\_{n-1}÷2$, where $a\_{1}=50$ for $n\geq 2$

3. $a\_{1}=2$ and $a\_{i+1}=2a\_{i}$, for $i\geq 1$,

4. $a\_{1}=3$ and $a\_{i+1}=3a\_{i}$, for $i\geq 1$,

$5. a\_{1}=4$ and $a\_{i+1}=4a\_{i}$, for $i\geq 1$,

$6. a\_{1}=1$ and $a\_{i+1}=(-1)a\_{i}$, for $i\geq 1$,

$7. a\_{1}=64$ and $a\_{i+1}=(-\frac{1}{2})a\_{i}$, for $i\geq 1$,

8. For each sequence below, an explicit formula is given. Write the first 5 terms of each sequence. Then, write a recursive formula for the sequence.

* 1. $a\_{n}= 2n+10$ for $n\geq 1$

* 1. $a\_{n}=\left(\frac{1}{2}\right)^{n-1}$for $n\geq 1$

9.For each sequence, write an explicit and recursive formula.

a. 1, $-$1, 1, $-$1, 1, $-$1, …

 b. $\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},…$

 c. It follows a “plus one” pattern: $8, 9, 10, 11, 12,$ ….

 d. It follows a “times 10” pattern: $4, 40, 400, 4000,$ ….

 e. Doug accepts a job where his starting salary will be $30,000 per year, and each year he will receive a raise of $3,000.

 f. A bacteria culture has an initial population of 10 bacteria, and each hour the population triples in size.