Lesson 2A: Geometric Sequences, Explicit Formula

Classwork

**Example 1**

Consider the sequence 5, 8, 11, 14, 17, ….

* 1. Write the recursive formula and the explicit formula for the sequence.
  2. Explain how each part of the formula relates to the sequence.

**Example 2**

Now consider the sequence 5, 15, 45, 135,…

a. What are the next three terms in the sequence?

Consider the sequence that follows a “multiplication of 3” pattern: ….

Consider the sequence that follows a “multiplication of pattern:

|  |  |
| --- | --- |
| An **geometric sequence** is a numerical pattern that increases  or decreases by multiplying the previous term by a nonzero constant called the **common ratio.** | |
| 1, -4, 16, -64, -256, …  r = -4 | 266, 128, 64, 32, … |

Determine whether each sequence is a geometric sequence.

a. -4, -2, -1, - , … b. 1, 4, 9, 25, ... c. d. -3, 15, -75, 375, …

Find the next three terms in the geometric sequence .

Find the next four terms of the geometric sequence -4, 2, -1, , … .

Each term in a geometric sequence can be expressed in terms of the **first term a1** and the **common ratio r**.

5, 15, 45, 135,…

|  |  |  |  |
| --- | --- | --- | --- |
| **Term** | **Symbol** | **In terms of a1 and d** | **Numbers** |
| first term | a1 | a1 | 5 |
| second term | a2 | a1r | 5(3) = |
| third term | a3 | a1r2 | 5(3)2 = |
| fourth term | a4 | a1r3 | 5(3)3 = |
| … | … | … | … |
| nth term | an | **a1r(n-1)** | 5r(n-1) |

Find the 15th term of the geometric sequence above.

Write an equation for the nth term of the geometric sequence

**Step 1: Find the common ratio.**

**Step 2: Write the equation.**

**an = a1r(n-1)**

Find the 9th term:

Write an equation for the nth term of the geometric sequence

**Step 1: Find the common ratio.**

**Step 2: Write the equation.**

**an = a1r(n-1)**

Find the 15th term:

Find the 25th term:

Write an equation for the nth term of the geometric sequence -4, 2, -1, , … .

**Step 1: Find the common ratio.**

**Step 2: Write the equation.**

**an = a1r(n-1)**

Find the 10th term:

Find the 20th term:

Lesson 2B: Geometric Sequences, Recursive Formula

Classwork

**Example 1**

What sequence does an+1 = an · 3 for and a1 = 5 generate?

What sequence does an+1 = an ÷ 3 for and a1 = 30 generate?

In a **geometric sequence** each term is found by multiplying a fixed number called the **common ratio(*r*)**, to the previous term. Example: 5, 15, 45, 135,…

The following equation is a recursive definition of a geometric sequence:

**an+1= an r**

|  |  |  |  |
| --- | --- | --- | --- |
| **Term** | **Symbol** | **In terms of a1 and d** | **Numbers** |
| first term | a1 | a1 | 5 |
| second term | a2 | a1 · r | 5 · 3 = |
| third term | a3 | a2 · r | 15 · 3 = |
| fourth term | a4 | a3 · r | 45 · 3 = |
| … | … | … | … |
| nth term | an | an-1 · r | an |
| nth + 1 term | an+1 | an · r | an+1 |

**Example 1**

**For problems 1-3,the recursive formula is given list the first five terms of each sequence.**

1. , where for

1. 6, where for

1. , where for

4. Write the recursive formula for the geometric sequence 64, 32, 16, 8, … , where a1 is 64. Find the next three terms.

5. Write the recursive formula for the geometric sequence -3, 15, -75, 275 … , where a1 is -3. Find the next four terms.

6. Determine the recursive formula for each of the following geometric sequences.

a. 24, 36, 54, 81, …. b. 9, 3, 1, , ….

**Example 2**

1. Consider a sequence given by the formula where .

* 1. List the first five terms of the sequence.
  2. Write an explicit formula.
  3. Find and of the sequence.

2. Graph the first five terms of the geometric sequence 64, 32, 16, 8, … ,

|  |  |
| --- | --- |
| **an** | **an+1 = an ÷ 2** |
| a1 | 64 |
| a2 | 32 |
| a3 | 16 |
| a4 | 8 |
| a5 |  |

[](http://www.google.com/imgres?imgurl&imgrefurl=http://www.crestviewlocal.k12.oh.us/chs/staff/mcc/cp10.html&h=0&w=0&sz=1&tbnid=r8ugDHFRf1kAzM&tbnh=225&tbnw=224&prev=/search?q=coordinate+plane&tbm=isch&tbo=u&zoom=1&q=coordinate%20plane&docid=bc5yn1XHt6D8NM&hl=en&ei=hny_Ua2aKumSiALt9YHoCw&ved=0CAEQsCU)

3. Graph the first five terms of the geometric sequence 3, 6, 12, 24,…

|  |  |
| --- | --- |
| **an** | **an+1 = an · 2** |
| a1 | 3 |
| a2 | 6 |
| a3 | 12 |
| a4 | 24 |
| a5 |  |

[](http://www.google.com/imgres?imgurl&imgrefurl=http://www.crestviewlocal.k12.oh.us/chs/staff/mcc/cp10.html&h=0&w=0&sz=1&tbnid=r8ugDHFRf1kAzM&tbnh=225&tbnw=224&prev=/search?q=coordinate+plane&tbm=isch&tbo=u&zoom=1&q=coordinate%20plane&docid=bc5yn1XHt6D8NM&hl=en&ei=hny_Ua2aKumSiALt9YHoCw&ved=0CAEQsCU)

4. Find the 11th term if the sequence 3, -6, 12, -24, … .

Lesson Summary

RECURSIVE SEQUENCE (description). An example of a *recursive sequence* is a sequence that (1) is defined by specifying the values of one or more initial terms and (2) has the property that the remaining terms satisfy a recursive formula that describes the value of a term based upon an expression in numbers, previous terms, or the index of the term.

An explicit formula specifies the th term of a sequence as an expression in .

A recursive formula specifies the th term of a sequence as an expression in the previous term (or previous couple of terms).

Problem Set 2B

For problems 1-7, list the first five terms of each sequence.

1. , where for
2. , where for

3. and , for ,

4. and , for ,

and , for ,

and , for ,

and , for ,

8. For each sequence below, an explicit formula is given. Write the first 5 terms of each sequence. Then, write a recursive formula for the sequence.

* 1. for

* 1. for

9.For each sequence, write an explicit and recursive formula.

a. 1, 1, 1, 1, 1, 1, …

b.

c. It follows a “plus one” pattern: ….

d. It follows a “times 10” pattern: ….

e. Doug accepts a job where his starting salary will be $30,000 per year, and each year he will receive a raise of $3,000.

f. A bacteria culture has an initial population of 10 bacteria, and each hour the population triples in size.